PHYSICAL REVIEW E **69**, 066411 (2004)

Dusty plasmas in a constant electric field: Role of the electron drag force

S. A. Khrapak* and G. E. Morfill
Centre for Interdisciplinary Plasma Science, Max-Planck-Institut für Extraterrestrische Physik, D-85741 Garching, Germany
(Received 25 January 2004; published 15 June 2004)

We investigate the forces experienced by a microparticle immersed in a weakly ionized plasma with constant electric field. These are electric force and the forces associated with the momentum transfer from electrons and ions drifting in the field (electron and ion drag forces). It is shown that the effect of the electron drag, which is often neglected, can be substantial in a certain parameter range. Numerical calculation of the forces for a reasonable set of plasma parameters is performed to illustrate the importance of this effect.

DOI: 10.1103/PhysRevE.69.066411 PACS number(s): 52.27.Lw, 52.20.−j

I. INTRODUCTION

Complex (dusty) plasmas consist of ions, electrons, highly charged micron-sized particles (dust grains), and neutral gas. In laboratory experiments the grain component can be easily observed, since the characteristic time scale is of the order of a fraction of a second. This allows us to investigate a variety of fundamental processes (phase transitions, transport, wave phenomena, etc.) at the most fundamental (kinetic) level. Not surprisingly, complex plasmas have received much attention in recent years.

One of the most fundamental issues concerning dust grains in plasmas is the question of the forces that the grains experience. The knowledge of the forces is essential when equilibrium states as well as transport properties of the grain component are considered. Depending on plasma conditions forces such as gravity, electric, ion and neutral drag, thermophoresis were demonstrated to play an important role [1–3].

The focus of the present paper is on the electron drag force F_e arising due to the momentum transfer from the electrons drifting relative to the charged grain. This force was often ignored previously: Based on the small electron mass it was assumed to be much weaker than the ion drag force F_i [4,5]. A rough estimate of the ratio of the ion to electron drag force in a quasineutral plasma can be obtained using the Coulomb scattering theory for pointlike grains and assuming that the Coulomb logarithms for electron-grain and ion-grain collisions are of the same order of magnitude. Then for subthermal drifts the force ratio is $F_i/F_e \sim (m_i/m_e)^{1/2}(T_e/T_i)^{3/2}(u_i/u_e)$, where $m_{e(i)}$ and $T_{e(i)}$ are the mass and the temperature of electrons (ions), and $u_{e(i)}$ denotes the relative drift velocity between electrons (ions) and the dust grain. When u_i and u_e are comparable (e.g., for grain motion in a stationary plasma background) the electron drag force is smaller than that due to ions if $(m_i/m_e)^{1/3} > T_i/T_e$ (a similar estimation can be found in Ref. [5]), which is always satisfied. However, when $u_e \gg u_i$, the usual practice to neglect electron drag is less obvious. This situation is, however, quite natural for a weakly ionized plasma in a constant electric field (e.g., the positive column of a dc gas discharge), where the drift velocities of electrons and ions are determined by the balance between the electric field force and momentum loss in collisions with neutrals. In this case $u_i/u_e \sim (m_e/m_i)^{1/2}(T_e/T_i)^{1/2}(\sigma_{en}/\sigma_{in})$, where σ_{en} and σ_{in} are the effective momentum transfer cross sections for electron-neutral and ion-neutral collisions. The force ratio is then $F_i/F_e \sim (T_e/T_i)^2(\sigma_{en}/\sigma_{in})$. In noble gases σ_{in} is typically more than one order of magnitude larger than σ_{en} and we see immediately that electron drag would dominate provided T_e/T_i is not too high. (In this context the noble gases argon, krypton, and xenon are of special interest since σ_{en} has a pronounced minimum for the electron energies of about 1 eV—the so called Ramsauer-Townsend effect [6].) The conclusion is, therefore, that the electron drag force cannot always be neglected.

In order to get further insight into the problem we consider details of the momentum transfer in electron-grain collisions. An accurate expression for the electron drag force in complex plasmas is then obtained. Using this result we compare the magnitude of the electron drag force with the electric and ion drag forces acting on a negatively charged dust grain in a weakly ionized plasma in an external electric field. It is shown that the electron drag force can dominate for a certain range of plasma parameters.

II. ELECTRON DRAG FORCE

Let us consider a dust grain immersed in a weakly ionized plasma with a constant electric field **E**. The electrons and ions drift in opposite directions with velocities

$$\mathbf{u}_e = -\frac{e\mathbf{E}}{m_e\nu_e}, \quad \mathbf{u}_i = \frac{e\mathbf{E}}{m_i\nu_i}, \tag{1}$$

where $\nu_{e(i)}$ is the effective collision frequency for electrons (ions). In a weakly ionized plasma collisions with neutrals play a dominant role. For sufficiently weak electric fields the drifts are subthermal. In this case $\nu_\alpha \simeq n_n\sigma_{\alpha n}v_{T_\alpha}$, where n_n is the neutral density and $v_{T_\alpha} = \sqrt{T_\alpha/m_\alpha}$ is the thermal velocity of either electrons ($\alpha=e$) or ions ($\alpha=i$). The ratio of the drift velocity to the thermal velocity is then $u_\alpha/v_{T_\alpha} \simeq eEl_\alpha/T_\alpha$, where $l_\alpha = 1/n_n\sigma_{\alpha n}$ denotes the mean free path. Weak electric fields corresponding to subthermal drifts ($u_\alpha \lesssim v_{T_\alpha}$) have to satisfy the inequality $E \lesssim \max\{T_e/el_e, T_i/el_i\}$.

*Electronic mail: skhrapak@mpe.mpg.de

A charged grain immersed in such a plasma is affected by (at least) three forces: Electric force $F_{\rm el}=QE$ (Q is the grain charge) and electron and ion drag forces. For a negatively charged grain $F_{\rm el}$ and F_e are directed opposite to the electric field direction, while F_i is parallel to the field. The competition between these forces determines transport of dust grains. Below we calculate the relationship between these forces for a range of plasma parameters.

The electron drag force is due to two processes: momentum transfer in direct collisions with a grain and momentum transfer from the electrons that are scattered in the electric field of the grain (but not collected). The first process is often referred to as the "collection" force while the second is often called "orbital" force. The orbital motion limited (OML) theory is applicable to describe electron collection. According to this theory an electron can undergo a direct collision with a spherical grain only if its impact parameter is sufficiently small, $\rho<\rho_c$, where

$$\rho_c(v)=a\begin{cases}\sqrt{1-\dfrac{2r_{\rm C}(v)}{a}}, & 2r_{\rm C}(v)/a<1\\ 0, & 2r_{\rm C}(v)/a>1\end{cases}\qquad(2)$$

is the maximum impact parameter for collection. Here $r_{\rm C}(v)=|Q|e/m_ev^2$ is the Coulomb radius for electron-grain collisions and a is the grain radius. Expression (2) shows that due to the repulsive character of the interaction potential only sufficiently energetic electrons can be collected. Assuming that the initial electron momentum is transferred to the grain, one obtains the momentum transfer cross section for electron collection,

$$\sigma_{\rm coll}(v)=\pi\rho_c^2(v),\qquad(3)$$

which is equal to the electron collection cross section.

To describe the electron scattering by the grain the standard Coulomb scattering approach can be used. It is applicable when $r_{\rm C}(v_{T_e})\ll\lambda_{\rm D}$, which is usually the case. This gives for the orbital momentum transfer cross section

$$\sigma_{\rm orb}(v)=2\pi r_{\rm C}^2(v)\ln\left[\frac{\lambda_{\rm D}^2+r_{\rm C}^2(v)}{\rho_c^2(v)+r_{\rm C}^2(v)}\right].\qquad(4)$$

In deriving Eq. (4) the integration over impact parameters was performed from $\rho_{\rm min}=\rho_c$ to $\rho_{\rm max}=\lambda_{\rm D}$ (see, e.g., Refs. [7,8]). Note that the screening length for the conditions we consider (low neutral gas pressures, subthermal drifts) is given by the linearized Debye radius, $\lambda_{\rm D}=(\lambda_{{\rm D}e}^{-2}+\lambda_{{\rm D}i}^{-2})^{-1/2}$ (for a recent discussion see, for example, Ref. [9]). For high neutral gas pressures Eqs. (2)–(4) are not directly applicable. Electrons are not collisionless and the screening can be quite different from weakly collisional case [10].

In order to derive the electron drag force the cross sections should be integrated over the electron velocity distribution function. We assume a shifted Maxwellian distribution (note, however, that other distributions are also possible in gas discharges), which in the case of subthermal electron drift can be written as $f(\mathbf{v})\simeq f_0(v)(1+\mathbf{uv}/v_{T_e}^2)$. The integration is complicated by the necessity to consider separately velocities satisfying $2r_{\rm C}(v)/a>1$ and $2r_{\rm C}(v)/a<1$, but otherwise it is straightforward. The final result can be presented in the form

$$F_e=(8\sqrt{2\pi}/3)a^2n_em_ev_{T_e}u_e\Phi_e(z,L),\qquad(5)$$

where n_e is the density of electrons and the factor $\Phi_e(z,L)$ accounts for the electron-grain electrostatic interaction ($\Phi_e=1$ for an uncharged grain). Here $z=|Q|e/aT_e$ is the dimensionless potential of the grain and $L=\lambda_{\rm D}/a$ is the ratio of the screening length to the grain radius. The contribution from direct collisions is $\Phi_e^{\rm Coll}(z)=[1+(z/2)]e^{-z}$. The contribution from scattering is $\Phi_e^{\rm Orb}(z,L)=\frac{1}{4}z^2\Lambda_{ed}$, where

$$\Lambda_{ed}=\int_0^\infty e^{-x}\ln\left(1+4L^2\frac{x^2}{z^2}\right)dx-2\int_z^\infty e^{-x}\ln\left(\frac{2x}{z}-1\right)dx\qquad(6)$$

is the Coulomb logarithm for electron-grain collisions (integrated over the Maxwellian distribution). In the limit $2L/z\gg1$ the integration gives

$$\Lambda_{ed}\simeq2\left[\ln\frac{2L}{z}-C+e^{-z/2}{\rm Ei}\left(-\frac{z}{2}\right)\right],\qquad(7)$$

where $C\simeq0.577$ is the Euler constant. The leading logarithmic term is dominant so that $\Lambda_{ed}\simeq2\ln(2L/z)$. This result improves on the estimation of the Coulomb logarithm $\Lambda_{ed}\sim2\ln L$ used in Ref. [11]. In Fig. 1 we show the value of the Coulomb logarithm calculated as a function of $a/\lambda_{\rm D}\equiv L^{-1}$ for three different values of z. The agreement between the direct numerical integration (6) and the approximate expression (7) is good as long as $2L/z\gtrsim1$. The simplified relation $\Lambda_{ed}\simeq2\ln L$ also agrees reasonably (within $\sim30\%$ accuracy) with the calculations. For very simple qualitative estimates one can use $\Lambda_{ed}\sim10$ for $a/\lambda_{\rm D}\sim10^{-2}$, independent of the value of z.

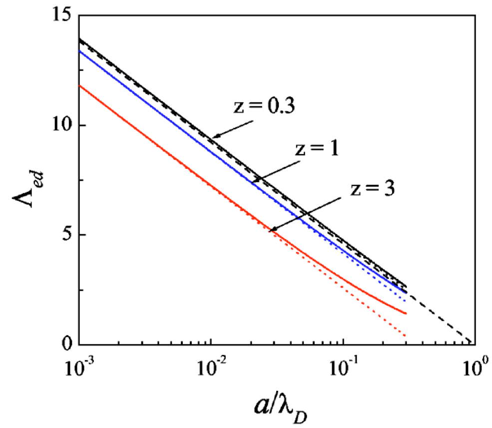

FIG. 1. The value of the Coulomb logarithm Λ_{ed} for electron-grain collisions as a function of the ratio of the grain radius to the plasma screening length, $a/\lambda_{\rm D}$, for three values of dimensionless grain potential $z=|Q|e/aT_e$. Solid lines correspond to the exact numerical integration of Eq. (6). Dotted lines show the approximate analytical expression (7). The dashed line corresponds to the estimate $\Lambda_{ed}\simeq2\ln L$.

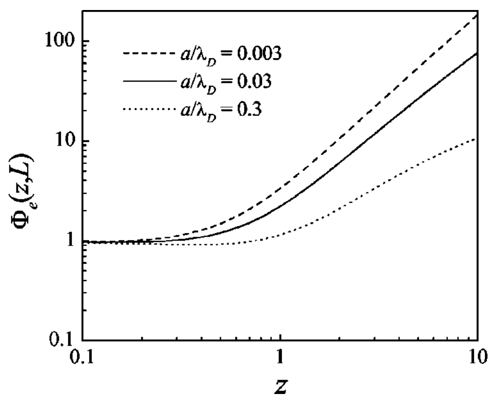

FIG. 2. The ratio of the momentum transfer in electron collisions with a charged grain to the momentum transfer in collisions with an uncharged grain, $\Phi_e(z,L)$, as a function of dimensionless grain potential $z=|Q|e/aT_e$. The calculations shown are for three values of the ratio of grain radius to the screening length $a/\lambda_D \equiv L^{-1}$. Note that scattering increases the momentum transfer considerably for $z \gtrsim 1$.

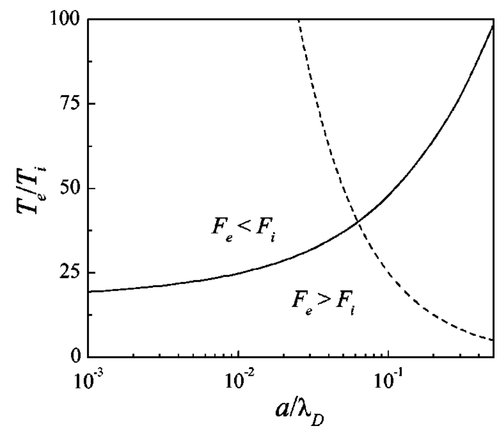

FIG. 3. The figure shows the contour of a constant ratio of the electron-to-ion drag forces, $F_i/F_e=1$, in the $(a/\lambda_D, \tau)$ plane for an argon plasma ($\sigma_{in}/\sigma_{en} \sim 200$ and $z \sim 2$). The dashed line corresponds approximately to the upper limit of the applicability of the theory used to calculate the ion drag force, $|Q|e/T_i\lambda_D \lesssim 5$ (see Ref. [7]).

Figure 2 shows the factor $\Phi_e(z,L)$ as a function of z for three different values of L. One can see that $\Phi_e(z,L) \sim 1$ for $z \lesssim 1$—the electrostatic interaction is not important in this regime. For $z \gtrsim 1$ the contribution from collection becomes exponentially small, but the contribution from elastic scattering grows with z. The latter leads to an overall increase in $\Phi_e(z,L)$ with z. Figure 2 shows that the smaller the grain the larger is the effect of electrostatic interaction.

In passing we note that the expression for the electron drag force [Eq. (5)] can be cast in the form $F_e = m_d \nu_{de} u_e$. Here m_d is the dust grain mass and ν_{de} is the characteristic momentum transfer rate in electron-grain collisions. From the momentum conservation we immediately obtain the momentum transfer frequency characterizing the momentum loss of the electron gas in collisions with the grain component, $\nu_{ed} = \nu_{de} n_d m_d / n_e m_e$, where n_d is the grain number density. The latter can be important for example when considering damping of Langmuir waves in dusty plasmas, the effect of dust grains on the plasma electrical conductivity, etc.

III. FORCE BALANCE

In this section we will compare the electron drag force with the electric and the ion drag forces for different plasma parameters. Let us first estimate the ratio of the electron to ion drag force, which will improve our estimations made in the introduction. For the ion drag force we use the recent approach of by Khrapak *et al.* [7,8], which is an extension of the standard Coulomb scattering theory. Similar to Eq. (5) the ion drag force can be written as

$$F_i = (8\sqrt{2\pi}/3) a^2 n_i m_i v_{T_i} u_i \Phi_i(z\tau, L), \qquad (8)$$

where n_i is the density of ions, $\tau = T_e/T_i$ is the electron-to-ion temperature ratio, and the factor $\Phi_i = \Phi_i^{\mathrm{Coll}} + \Phi_i^{\mathrm{Orb}}$ accounts for ion collection and scattering, similar to electron-grain collisions. The corresponding contributions are $\Phi_i^{\mathrm{Coll}}(z\tau) = 1 + (z\tau/2)$ and $\Phi_i^{\mathrm{Orb}}(z\tau, L) = (z^2\tau^2/4)\Lambda_{id}$, where $\Lambda_{id} = 2\int_0^\infty e^{-x} \ln[(2Lx + z\tau)/(2x + z\tau)]dx$ is the modified Coulomb logarithm for ion-grain collisions integrated over the Maxwellian velocity distribution function for ions. Equation (8) is identical to Eqs. (11) and (12) of Ref. [7]. The reason for the modification of the Coulomb logarithm is due to the fact that the Coulomb radius for ion-grain collisions is often greater or comparable to the plasma screening length, which makes the standard Coulomb scattering theory inapplicable. From Eqs. (5) and (8) assuming $n_e \simeq n_i$ and taking into account Eq. (1) we get

$$\frac{F_e}{F_i} \simeq \frac{\sigma_{in}}{\sigma_{en}} \frac{\Phi_e(z,L)}{\Phi_i(z\tau, L)}.$$

This expression implies that the ratio F_e/F_i depends on the gas composition (which determines the ratio σ_{in}/σ_{en}) and three parameters z, τ, and L. An example calculation of the ratio F_e/F_i is shown in Fig. 3. The solid line marks the balance $F_e = F_i$ in the $(a/\lambda_D, \tau)$ plane for an argon plasma. The following parameters were fixed for our calculations: argon plasma with effective cross section for ion and electron collisions with neutrals $\sigma_{in} \simeq 2 \times 10^{-14}$ cm^2 and $\sigma_{en} \simeq 1 \times 10^{-16}$ cm^2 (for $T_e \sim 1$ eV) [12], and dimensionless grain potential $z \simeq 2$ (this value follows from the OML theory for $\tau \sim 100$). Figure 3 shows that the electron drag force dominates over the ion drag force for $T_e/T_i \lesssim 25$, in agreement with the arguments given in the Introduction. The importance of the electron drag increases with increasing grain size. Our calculations are applicable below the dashed line, which corresponds to the upper limit of the ion drag model applicability (see Ref. [7]). We note that the parameter range, where electron drag dominates over ion drag, seems not to be exotic for dusty plasma experiments in dc discharges. For example, typical glow discharge conditions $T_e \sim 1$ eV, $T_i \sim 0.1$ eV for an argon plasma at neutral gas pressure $p = 0.1$ Torr were reported in Ref. [13].

The relationships among F_e, F_i, and F_{el} depend on more parameters, especially when the ion drag model of Eq. (8)

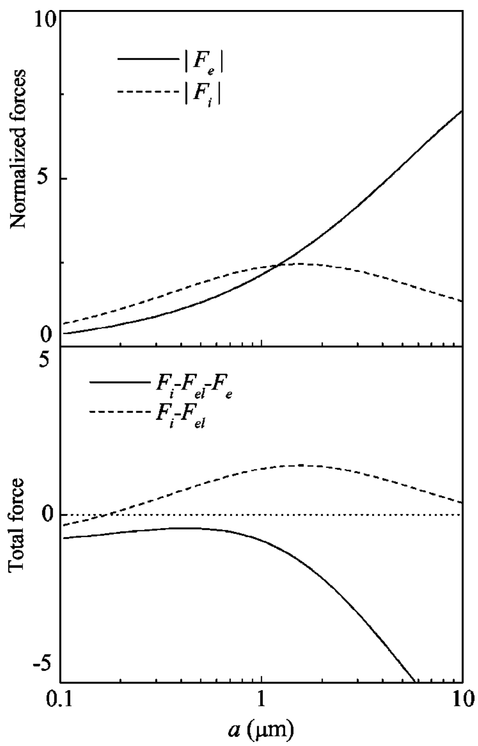

FIG. 4. (a) The magnitudes of the electron (F_e) and ion (F_i) drag forces normalized to the electric force $F_{\rm el}$ as functions of grain radius a. (b) The sum of the forces $F_i - F_{\rm el} - F_e$ normalized to the electric force as a function of a. The dotted line shows the (incorrect) result which would be obtained by neglecting the electron drag force. For the plasma parameters see text.

breaks down. For this reason, and in order to illustrate the importance of the electron drag force, we took a certain set of realistic plasma parameters and calculated the forces numerically. We again consider an argon plasma at pressure $p \sim 0.1$ Torr ($n_n \sim 3\times 10^{15}$ cm^{-3}) and plasma number density $n_e \simeq n_i \simeq 2\times 10^9$ cm^{-3}. Electron and ion temperatures are $T_e \simeq 1$ eV and $T_i \simeq 0.03$ eV ($\tau = 33$). Note that for these parameters the assumption of subthermal drifts limits our calculations to rather small electric fields, $E \lesssim 0.3$ V/cm.

Figure 4(a) shows the dependence of the absolute magnitude of the electron and ion drag forces on grain radius. Here the ion drag force was calculated using an approximate expression for the ion-grain momentum transfer cross section proposed in Ref. [14]. This approach is more accurate for the case of "strong" interaction between ions and grain [15], which occurs at $a \gtrsim 1$ μm for the chosen plasma parameters. The forces are normalized to the electric force. According to Fig. 4(a) the electron and the ion drag forces are comparable and both these forces dominate over the electric force. Moreover, $F_e > F_i$ for $a \gtrsim 1$ μm. This indicates the important role which the electron drag force can play in dusty plasmas. Figure 4(b) illustrates qualitative changes arising when taking into account the electron drag force. If one neglects F_e, then the grain would move in the direction of the electric field because $F_i > F_{\rm el}$ (except for very small submicron grains). Inclusion of the electron drag force reverses the grain motion for the considered plasma parameters.

Finally, let us briefly discuss the parameter range where the electron drag force can play a substantial role. First, a significant electron drift should be present, which implies that the force exerted on electrons by the electric field is compensated by their momentum loss in collisions, but not by the inhomogeneity in electron density ("pressure" term) in the equation of motion for electrons. Relevant example is a longitudinal electron drift in a positive column of a dc discharge. At the same time the considered effect vanishes in complex plasmas occurring in the bulk of rf discharges. Next, the electron drag force is very sensitive to the electron temperature. There are two reasons for that (i) $F_e/F_i \propto (T_i/T_e)^2$, and $F_e/F_{\rm el} \propto T_i/T_e$; (ii) for most noble gases σ_{en} is an increasing function of T_e (in the range $1\ {\rm eV} \lesssim T_e \lesssim 10$ eV) and hence $F_e \propto \sigma_{en}^{-1}$ decreases with T_e. As pointed out in the introduction the effect can be especially important in argon, krypton, and xenon due to the Ramsauer-Townsend effect, and also in neon where σ_{en} is also relatively small. Increasing the neutral gas pressure and/or decreasing the ionization fraction would lower the role of electron and ion drag forces compared to the electrostatic force. Also the electron drag force increases with grain size as shown in Fig. 4. Summarizing, we expect electron drag to be important for micron (and larger) size grains in low pressure dc gas discharges in most of the noble gases with relatively small electron temperature ($T_e \lesssim 1$ eV).

The obtained results have to be taken into account in planned experiments with dusty plasma in a dc discharge under microgravity conditions (PK-4 experiment). Here the gravity is absent and the most important forces acting on dust grains are the electrostatic, ion and electron drags and the neutral drag force (if a gas flow and/or grain motion is present). As shown in the present paper the electron drag force can have a significant effect under certain (not too exotic) conditions.

IV. CONCLUSIONS

We have studied the role of the electron drag force in a weakly ionized plasma in the presence of a constant electric field—a common situation for the positive column of a dc discharge. Using the accurate expression for the electron drag force obtained in this paper we were able to show that electron drag can be a dominant force for certain plasma parameters. The regimes where this can occur were identified.

ACKNOWLEDGMENT

This work was partly supported by DLR under Grants Nos. 50WM9852 (PKE-Nefedov) and 50WP0204 (PK-4).

[1] M. S. Barnes, J. H. Keller, J. C. Forster, J. A. O'Neill, and D. K. Coultas, Phys. Rev. Lett. **68**, 313 (1992).
[2] J. E. Daugherty and D. B. Graves, J. Appl. Phys. **78**, 2279 (1995).
[3] H. Rothermel, T. Hagl, G. E. Morfill, M. H. Thoma, and H. M. Thomas, Phys. Rev. Lett. **89**, 175001 (2002).
[4] T. G. Northrop and T. J. Birmingham, Planet. Space Sci. **38**, 319 (1990).
[5] G. E. Morfill, V. N. Tsytovich, and H. Thomas, Plasma Phys. Rep. **29**, 1 (2003).
[6] H. S. W. Massey, E. H. S. Burhop, and H. B. Gilbody, *Electronic and Ionic Impact Phenomena* (Oxford University Press, New York, 1969).
[7] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, and H. M. Thomas, Phys. Rev. E **66**, 046414 (2002).
[8] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, and H. M. Thomas, in *Dusty Plasmas in the New Millenium*, edited by R. Bharuthram, M. A. Hellberg, P. K. Shukla, and F. Verheest, AIP Conf. Proc. No. 649 (AIP, Melville, NY, 2002), p. 341.
[9] S. A. Khrapak *et al.*, Phys. Plasmas **10**, 4579 (2003).
[10] O. Bystrenko and A. Zagorodny, Phys. Rev. E **67**, 066403 (2003).
[11] S. Benkadda, P. Gabbai, V. N. Tsytovich, and A. Verga, Phys. Rev. E **53**, 2717 (1996).
[12] A. Gilardini, *Low Energy Electron Collisions in Gases* (Wiley, New York, 1972); *Physical quntities: Reference book*, edited by I. S. Grigor'ev and E. Z. Meilikhov (Energoatomizdat, Moscow, 1991) (in russian)
[13] S. Takamura, T. Misawa, N. Ohno, S. Nunomura, M. Sawai, K. Asano, and P. K. Kaw, Phys. Plasmas **8**, 1886 (2001).
[14] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, S. K. Zhdanov, and H. M. Thomas, IEEE Trans. Plasma Sci. (to be published).
[15] S. A. Khrapak, A. V. Ivlev, G. E. Morfill, and S. K. Zhdanov, Phys. Rev. Lett. **90**, 225002 (2003).